

CHEMICAL DYNAMICS IN CONDENSED PHASES

Relaxation, Transfer and Reactions in Condensed Molecular systems

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